Resource Calendaring for Mobile Edge Computing in 5G Networks

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Overview: This technical report provides additional material for our submission to an IEEE conference. In detail, Section I presents the reformulation of the optimization model $\mathcal{P}0$ in the main contents of our submitted paper, and Section II shows the full set of numerical results.

I. Problem Reformulation

Problem $\mathcal{P}0$ formulated in our submitted paper cannot be solved directly and efficiently due to following reasons:

- We perform optimal routing (the routing path $\mathcal{R}^{kv}$ is a variable in our model, since many paths may exist from each request source node s^k to a generic node v in the network); furthermore, we must ensure that the properties of no-splitting, continuity and acyclicity are respected for our routing solution.
- Variables $\mathcal{R}^{kv}$ and q^{kv} are "intertwined": to find the optimal routing, the percentage of request processed at each node v should be known, and at the same time, to solve the optimal allocation for a request, the routing path should be known.
- $\mathcal{P}0$ contains indicator functions and constraints, e.g. (4), (6), (11), etc., which cannot be directly and easily processed by most solvers.

To deal with these challenging issues, we propose an equivalent reformulation of $\mathcal{P}0$, which can be solved very efficiently with the Branch and Bound method. Moreover, based on the reformulated problem, we propose an heuristic algorithm which can get near-optimal solutions in a short computing time.

A. Link Latency

As we stated before, to compute the link latency, we need to determine the routing path $\mathcal{R}^{kv}$, and this issue will be specifically handled in subsection I-C. Assuming $\mathcal{R}^{kv}$ has been determined, we first introduce a binary variable γ_e^{kv} defined as follows:

$$\gamma_e^{kv} = \begin{cases} 1, & \text{if } e \in \mathcal{R}^{kv}, \\ 0, & \text{otherwise}, \end{cases} \quad \forall k, \forall v, \forall e,$$

which indicates whether e is used in the routing path $\mathcal{R}^{kv}$ or not. Note that only if request k is processed on node v (i.e., $b^{kv} = 1$) and $v \neq s^k$, the corresponding routing path is defined. Then we have:

$$\begin{cases} \gamma_e^{ks^k} = 0, & \forall k, \forall e, \\ \gamma_e^{kv} \leqslant b^{kv}, & \forall k, \forall v, \forall e. \end{cases} \tag{12}$$

Based on the above definitions, we can rewrite constraint (5) as:

$$\begin{cases} q^{kv}\lambda^k - (1-\gamma_e^{kv})\Lambda^k < p_e^{kv} B_e, \\ p_e^{kv} \leqslant \gamma_e^{kv}, \end{cases} \quad \forall k, \forall v, \forall e, \tag{13}$$

where $\Lambda^k = \lambda^k + c$ and $c = 1$ is a constant. Note that the term $(1-\gamma_e^{kv})$ permits to implement condition $e \in \mathcal{R}^{kv}$ in Eq. (5).

We now introduce variable h_e^{kv}, defined as follows:

$$h_e^{kv} = \frac{1}{p_e^{kv} B_e - q^{kv}\lambda^k + (1-\gamma_e^{kv})\Lambda^k}, \ \forall k, \forall v, \forall e. \tag{14}$$

This permits to transform Eq. (4) as $T_L^{kv} = \sum_{e\in\mathcal{E}} \gamma_e^{kv} h_e^{kv}$. We then need to linearize the product of the binary variable γ_e^{kv} and the continuous variable h_e^{kv}, and to this aim we introduce an auxiliary variable $g_e^{kv} = \gamma_e^{kv} h_e^{kv}$, thus also eliminate T_L^{kv}.

We first compute the value range of h_e^{kv} by considering the two cases: if $\gamma_e^{kv} = 0$, the range is $[(\Lambda^k)^{-1}, c^{-1}]$, where $c = \Lambda^k - \lambda^k$, and if $\gamma_e^{kv} = 1$, the range is $[B_e^{-1}, ((\beta^k - \alpha^k - d^k)\tau)^{-1}]$ based on constraint (2). In detail, if $\gamma_e^{kv} = 0$, then $p_e^{kv} = 0$ and the denominator of h_e^{kv} becomes $\Lambda^k - q^{kv}\lambda^k$, considering $q^{kv} \in [0, 1]$, the range of h_e^{kv} is computed as $[(\Lambda^k)^{-1}, c^{-1}]$; if $\gamma_e^{kv} = 1$, the denominator of h_e^{kv} becomes $p^{kv} B_e - q^{kv}\lambda^k$, therefore, the upper limit of the denominator is B_e. Given that h_e^{kv} represents the single link latency which must be less than the allowed maximum latency $(\beta^k - \alpha^k - d^k)\tau$, therefore, the range of h_e^{kv} is $[B_e^{-1}, ((\beta^k - \alpha^k - d^k)\tau)^{-1}]$. Then, the linearization is performed by the following constraints.

$$\begin{cases} \gamma_e^{kv} B_e^{-1} \leqslant g_e^{kv} \leqslant \gamma_e^{kv}((\beta^k - \alpha^k - d^k)\tau)^{-1}, \\ (1-\gamma_e^{kv})(\Lambda^k)^{-1} \leqslant h_e^{kv} - g_e^{kv} \leqslant (1-\gamma_e^{kv})c^{-1}. \end{cases} \tag{15}$$

At the same time, the link latency is rewritten as:

$$T_L^{kv} = \sum\nolimits_{e\in\mathcal{E}} g_e^{kv}.$$

Since $p_e^{kvt} = \delta^{kvt} p_e^{kv}$ is the product of binary and continuous variables, we linearize it as:

$$\begin{cases} 0 \leqslant p_e^{kvt} \leqslant \delta^{kvt}, \\ 0 \leqslant p_e^{kv} - p_e^{kvt} \leqslant 1 - \delta^{kvt}, \end{cases} \quad \forall k, \forall v, \forall t, \forall e. \tag{16}$$

Please remind that δ^{kvt} is a binary variable which is equal to 1 if $\xi_\star^k \leqslant t < \xi_\star^k + d^k + \left\lceil \frac{T_L^{kv}}{\tau} \right\rceil$, and 0 otherwise. As we can see both upper and lower bounds of t are variables. We reformulate δ^{kvt} by the following constraints:

$$\begin{cases} \xi_\star^k - t \leqslant (\beta^k - d^k)(1-\delta^{kvt}), \\ t - (\xi_\star^k + d^k + \pi_L^{kv}) < (\mathcal{T}_m - d^k + 1)(1-\delta^{kvt}), \\ 0 \leqslant \sum_{t' \in \mathcal{T}} \delta^{kvt'} - (d^k + \pi_L^{kv}) \leqslant (\beta^k - \alpha^k)(1-b^{kv}), \\ 0 \leqslant \pi_L^{kv} - \frac{T_L^{kv}}{\tau} < 1, \end{cases} \quad \forall k, \forall v, \forall t, \quad (17)$$

where π_L^{kv} is an auxiliary integer variable for expanding the ceil operation over $\frac{T_L^{kv}}{\tau}$. The first and the second inequalities respectively enforce $\delta^{kvt} = 0$, when $t < \xi_\star^k$ and $t \geqslant (\xi_\star^k + d^k + \pi_L^{kv})$, which is the ending time of the link transmission. The third one enforces $\delta^{kvt} = 1$ when t is in the range $[\xi_\star^k, \xi_\star^k + d^k + \left\lceil \frac{T_L^{kv}}{\tau} \right\rceil)$ and $b^{kv} = 1$.

B. Processing Latency and Storage Provisioning

Equation (7) is a nonlinear indicator function of the variables r^{kv} and q^{kv}. To handle this issue, we first introduce an auxiliary variable b^{kv} to indicate whether request k is processed on node v. According to the definition of q^{kv}, we have the following constraint:

$$q^{kv} \leqslant b^{kv} \leqslant M q^{kv}, \ \forall k, \forall v, \quad (18)$$

where $M > 0$ is a big value and such constraint implies that if $q^{kv} = 0$, the request k is not processed on node v, i.e. $b^{kv} = 0$. Based on the above, we can rewrite constraint (8) as:

$$\begin{cases} \eta^k q^{kv} \lambda^k - (1 - b^{kv}) < r^{kv} D_v, \\ r^{kv} \leqslant b^{kv}, \end{cases} \quad \forall k, \forall v. \quad (19)$$

Note that the term $(1 - b^{kv})$ permits to implement condition $q^{kv} > 0$ in Eq. (8). The storage capacity constraint (10) can be rewritten as follows:

$$\sum_{k \in \mathcal{K}} m^k b^{kv} \leqslant S_v, \ \forall v. \quad (20)$$

In equation (7), we observe that if $b^{kv} = 1$, we have:

$$\frac{1}{r^{kv} D_v - \eta^k q^{kv} \lambda^k} > \frac{1}{D_v} \geqslant \frac{1}{D_{max}},$$

where $D_{max} = \max_{v \in \mathcal{V}} D_v$, otherwise $r^{kv} D_v - \eta^k q^{kv} \lambda^k = 0$ resulting in $T_P^{kv} \to \infty$. To handle this case, we first define a new variable $T_{P'}^{kv}$ as follows:

$$T_{P'}^{kv} = \frac{1}{r^{kv} D_v - \eta^k q^{kv} \lambda^k + (1 - b^{kv}) D_{max}}. \quad (21)$$

From this equation, we have $b^{kv} = 1 \Rightarrow T_{P'}^{kv} = T_P^{kv} > \frac{1}{D_{max}}$ and $b^{kv} = 0 \Rightarrow T_{P'}^{kv} = \frac{1}{D_{max}}$, $T_P^{kv} = 0$. More in detail, this indicates that if a request k is accepted and processed on a set of nodes $\mathcal{V}_{sub} \subseteq \mathcal{V}$ (i.e., $b^{kv} = 1, v \in \mathcal{V}_{sub}$), the processing latency is determined by $\max_{v \in \mathcal{V}_{sub}} T_P^{kv} > \frac{1}{D_{max}}$, therefore, $T_{P'}^{kv}$ satisfies the related constraints and represents the exact processing latency when request k is accepted. Instead if k is rejected, then we have $z^{kt} = 0, b^{kv} = 0, \forall v, t$. Based on constraint (2) specifying that the ending time depends on the maximum latency and considering that a rational and meaningful request should satisfy $d^k + \left\lceil \frac{1}{D_{max} \tau} \right\rceil < \beta^k - \alpha^k$, we have $\xi_o^{kv} = d^k + \left\lceil \frac{T_{P'}^{kv}}{\tau} \right\rceil < \beta^k$. Therefore, $T_{P'}^{kv}$ is a valid representation for the processing latency and the reformulation has no influence on the solution of the optimization problem.

Since $r^{kvt} = \rho^{kvt} r^{kv}$ is the product of binary and continuous variables, we linearize it as:

$$\begin{cases} 0 \leqslant r^{kvt} \leqslant \rho^{kvt}, \\ 0 \leqslant r^{kv} - r^{kvt} \leqslant 1 - \rho^{kvt}, \end{cases} \quad \forall k, \forall v, \forall t. \quad (22)$$

Recall that ρ^{kvt} is a binary variable which is equal to 1 if $\xi_\star^k + \left\lceil \frac{T_L^{kv}}{\tau} \right\rceil \leqslant t < \xi_o^{kv}$, and 0, otherwise. We can see in ρ^{kvt} that both upper and lower bounds of t are variables. We reformulate ρ^{kvt} by the following constraints: The derivation is very similar to the one for δ^{kvt} (see inequality (17)) due to the similar definitions of the variables.

$$\begin{cases} \xi_\star^k + \pi_L^{kv} - t \leqslant (\beta^k - d^k)(1-\rho^{kvt}), \\ t - \xi_o^{kv} < (\mathcal{T}_m - d^k + 1)(1-\rho^{kvt}), \\ 0 \leqslant \sum_{t' \in \mathcal{T}} \rho^{kvt'} - (d^k + \pi_{P'}^{kv}) \leqslant (\beta^k - \alpha^k)(1-b^{kv}), \\ 0 \leqslant \pi_{P'}^{kv} - \frac{T_{P'}^{kv}}{\tau} < 1, \end{cases} \quad \forall k, \forall v, \forall t, \quad (23)$$

where $\pi_{P'}^{kv}$ is an auxiliary integer variable for expanding the ceil operation over $\frac{T_{P'}^{kv}}{\tau}$. Based on above, the deadline constraint (2) can be rewritten as:

$$\xi_\star^k + d^k + \pi_L^{kv} + \pi_{P'}^{kv} \leqslant \beta^k, \ \forall k, \forall v. \quad (24)$$

C. Routing Path

Based on the definitions introduced in the previous subsection, the traffic flow f_e^k can be transformed as:

$$f_e^k = \sum_{v \in \mathcal{V}} \gamma_e^{kv} q^{kv}, \forall k, \forall e. \quad (25)$$

Now we need to simplify the traffic flow conservation constraint (see Eq. (3)). To this aim, and to simplify notation, we first introduce in the network topology a "dummy" entry node 0 which connects to all source nodes $s^k, k \in \mathcal{K}$. All requests are coming through this dummy node and going to each source node with volume λ^k, i.e. $f_e^k = 1, \forall k, \forall e \in \mathcal{F}$, where $\mathcal{F} = \{(0, s^k) \,|\, k \in \mathcal{K}\}$ is the dummy link set. Then, we extend the definition of Φ_v^- to $\Phi_v^- = \{(v', v) \in \mathcal{E} \cup \mathcal{F}\}$. Equation (3) is hence transformed as:

$$\sum_{e \in \Phi_v^-} f_e^k - \sum_{e \in \Phi_v^+} f_e^k = q^{kv}, \ \forall k, \forall v. \quad (26)$$

Correspondingly, we add the following constraints to the set $\mathcal{F}$ of dummy links:

$$\gamma_e^{kv} = \begin{cases} b^{kv}, & \text{if } e = (0, s^k) \\ 0, & \text{otherwise}, \end{cases} \quad \forall k, \forall v, \forall e \in \mathcal{F}. \quad (27)$$

The final stage of our procedure is the definition of the constraints that guarantee all desirable properties that a routing path must respect: the fact that a *single path* is used (a request piece is no more splittable), the flow conservation constraints that provide *continuity* to the chosen path, and finally the absence of *cycles* in the routing path $\mathcal{R}^{kv}$. We would like to highlight that the request k can be only split at source node s^k, and each portion of such traffic is destined to an edge node v, and this is the reason why we have multiple routing paths $\mathcal{R}^{kv}, v \in \{1, 2, \cdots\}$.

To this aim, we introduce the following conditions:

- For an arbitrary node v', the number of incoming links used by a path $\mathcal{R}^{kv}$ is one, and thus variables γ_e^{kv} should satisfy the following condition:

$$\sum\nolimits_{e\in\Phi_{v'}^{-}} \gamma_e^{kv} \leqslant 1, \ \forall k, \forall v, \forall v'. \tag{28}$$

- The flow conservation constraint (see Eq. (26)) implements the continuity of a traffic flow.
- Every routing path should have an end or a destination to avoid loops. This can be ensured by the following equation:

$$\gamma_{(v,v')}^{kv} = 0, \ \forall k, \forall (v, v') \in \mathcal{E}. \tag{29}$$

Satisfying them along with the constraints illustrated before can guarantee that such properties of the routing path are respected. The proof is as follows:

Proof. a) Substitute Eq. (25) into (26) and make the following transformation:

$$\sum_{v'\in\mathcal{V}} q^{kv'}\Big(\sum_{e\in\Phi_v^{-}} \gamma_e^{kv'} - \sum_{e\in\Phi_v^{+}} \gamma_e^{kv'}\Big) = q^{kv}, \ \forall k, \forall v. \tag{30}$$

b) Based on constraints (12) and (27), we have:

$$\text{if } q^{kv'} = 0, \ \text{then} \ \sum_{e\in\Phi_v^{-}} \gamma_e^{kv'} - \sum_{e\in\Phi_v^{+}} \gamma_e^{kv'} = 0.$$

c) From a) and b), we have:

$$\begin{cases} \displaystyle\sum_{e\in\Phi_v^{-}} \gamma_e^{kv} - \sum_{e\in\Phi_v^{+}} \gamma_e^{kv} = 1, \forall k, \forall v \mid q^{kv} > 0, \\ \displaystyle\sum_{e\in\Phi_v^{-}} \gamma_e^{kv'} - \sum_{e\in\Phi_v^{+}} \gamma_e^{kv'} = 0, \forall k, \forall v, \forall v' \neq v. \end{cases}$$

d) Based on c), constraint (27), conditions (28) and (29) can be written as:

$$\begin{cases} \displaystyle\sum_{e\in\Phi_{s^k}^{-}} \gamma_e^{kv} = 1, \ \forall k, \forall v \mid q^{kv} > 0, & (31) \\ \displaystyle\sum_{e\in\Phi_v^{-}} \gamma_e^{kv} = 1, \ \forall k, \forall v \mid q^{kv} > 0, & (32) \\ \displaystyle\sum_{e\in\Phi_v^{-}} \gamma_e^{kv'} = \sum_{e\in\Phi_v^{+}} \gamma_e^{kv'} \leqslant 1, \ \forall k, \forall v, \forall v' \neq v. & (33) \end{cases}$$

Their practical meaning is explained as follows:

- (31) ensures dummy link $(0, s^k)$ to be the zeroth link in any routing path $\mathcal{R}^{kv}$ if $q^{kv} > 0$,
- (32) ensures node v to be the end node of the last link in any routing path $\mathcal{R}^{kv}$ if $q^{kv} > 0$,
- (33) ensures that if $v \in \mathcal{E}\backslash\{v'\}$ is an intermediate node in a routing path $\mathcal{R}^{kv'}$, v should have only one incoming link and one outgoing link. It also indicates the continuity of a request flow.

e) Given a non-empty routing path $\mathcal{R}^{kv'}$ ($q^{kv'} > 0$), check its validity by using the above conditions:

- Let $v = s^k$ in (33), then based on (31), $\sum_{e\in\Phi_v^{+}} \gamma_e^{kv'} = 1$. Next, we assume $e_1 = (s^k, v_1)$ is the first link of the routing path $\mathcal{R}^{kv'}$, then $\gamma_{e_1}^{kv'} = 1$;
- If $v_1 = v'$, then the path is found, otherwise, we continue with the following steps:
- Let $v = v_1$ in (33), due to $\gamma_{e_1}^{kv'} = 1$, $\sum_{e\in\Phi_{v_1}^{+}} \gamma_e^{kv'} = 1$. Next, we assume $e_2 = (v_1, v_2)$ is the second link of $\mathcal{R}^{kv'}$, then $\gamma_{e_2}^{kv'} = 1$;
- We continue to check the path following the way as above the two steps until the final target $v_n = v'$ is reached, along the whole path $(s^k \to v') = (e_1, e_2, \cdots, e_n)$.

Thus, if all the conditions are satisfied, $\mathcal{R}^{kv'}$ must be a valid routing path having the three properties. □

Based on the above reformulation of routing, the flow conservation constraints can be further improved as follows:

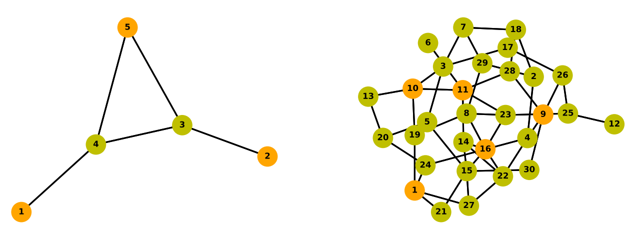
$$\begin{cases} \displaystyle\sum_{e\in\Phi_v^{-}} \gamma_e^{kv} = b^{kv}, \qquad \forall k, \forall v, & (34) \\ \displaystyle\sum_{e\in\Phi_{v'}^{-}} \gamma_e^{kv} = \sum_{e\in\Phi_{v'}^{+}} \gamma_e^{kv}, \forall k, \forall v, \forall v' \neq v. & (35) \end{cases}$$

D. Final Reformulated Problem $\mathcal{P}1$

After using the above reformulated constraints, we replace the corresponding original ones in $\mathcal{P}0$; the new reformulated problem is referred to as $\mathcal{P}1$, and is equivalent to $\mathcal{P}0$. Since constraints (14) and (21) are quadratic while the others are linear, $\mathcal{P}1$ is a mixed-integer quadratically constrained programming (MIQCP) problem, for which commercial and freely available solvers can be used, as we discussed in the numerical evaluation section.

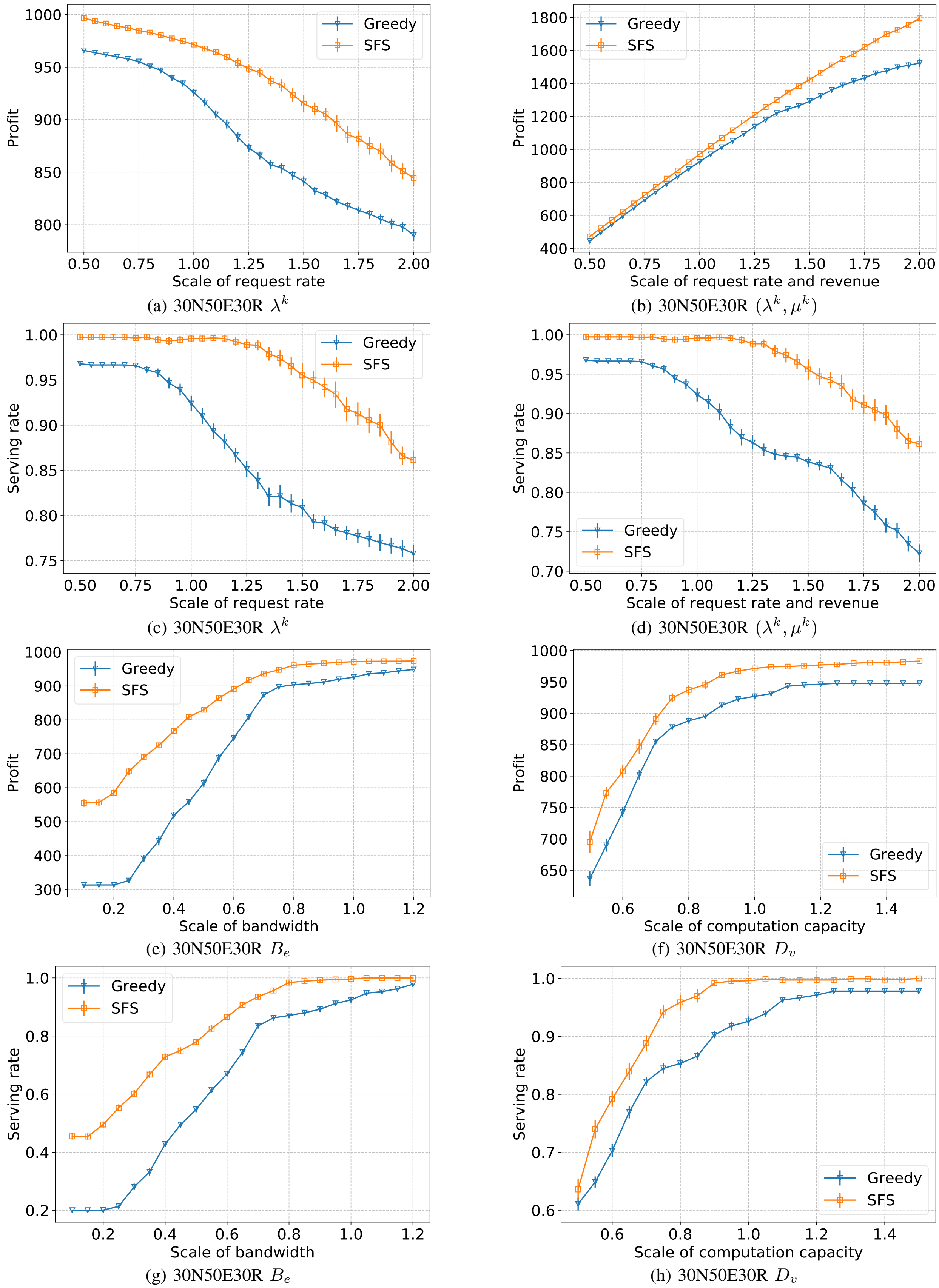
II. Numerical Results (Full)

This section mainly contains the network topologies (see Figure 1) used in our experiments and the corresponding results showing the effects of the considered network parameters (see Figure 2).



(a) 5N5E3R (b) 30N50E30R

Fig. 1: Network topologies. Ingress nodes for each graph are colored in orange. In scenario (a), each ingress node has one request, and in scenario (b), each ingress node has 6 different requests.



(a) 30N50E30R λ^k
(b) 30N50E30R (λ^k, μ^k)
(c) 30N50E30R λ^k
(d) 30N50E30R (λ^k, μ^k)
(e) 30N50E30R B_e
(f) 30N50E30R D_v
(g) 30N50E30R B_e
(h) 30N50E30R D_v

Fig. 2: Scenario 30N50E30R: profit and serving rate against scaling parameters λ^k, (λ^k, μ^k), B_e and D_v.